AGS 3 Name: ______________________ Period: _____ Date: ___________

Notes 3.9 – End Behavior

Warmup – Find all the roots.

1. $f(x) = 2x^3 + 3x^2 - 39x - 20 \quad root = 4$

4	2	3	−39	−20
		8	44	20
	2	11	5	0

$2x^2 + 11x + 5$

$(2x+1)(x+5)$

$\boxed{x = 4, -\frac{1}{2}, -5}$

2. $f(x) = x^4 - 4x^3 + 8x - 32 \quad factor: (x-4)$ $(x+2)$

4	1	−4	0	8	−32
		4	0	0	32
−2	1	0	0	8	0
		−2	4	−8	
	1	−2	4	0	

$x^2 - 2x + 4$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$

$x = \frac{2 \pm \sqrt{-12}}{2} = \frac{2}{2} \pm \frac{2i\sqrt{3}}{2}$

$\boxed{x = -2, 4, 1 \pm i\sqrt{3}}$

Multiply each pair of polynomials.

3. $(x+5)^2 \rightarrow (x+5)(x+5)$

$x^2 + 5x + 5x + 25$

$x^2 + 10x + 25$

4. $(x+\sqrt{3})(x-\sqrt{3})$

$x^2 - x\sqrt{3} + x\sqrt{3} - 3$

$\boxed{x^2 - 3}$

5. $(x-3)(x^2+3x+9)$

$\boxed{x^3 - 27}$

	x^2	$3x$	9
x	x^3	$3x^2$	$9x$
-3	$-3x^2$	$-9x$	-27

6. $(a+b)(a^2 - ab + b^2)$

$\boxed{a^3 + b^3}$

	a^2	$-ab$	b^2
a	a^3	$-a^2b$	ab^2
b	a^2b	$-ab^2$	b^3

Investigation

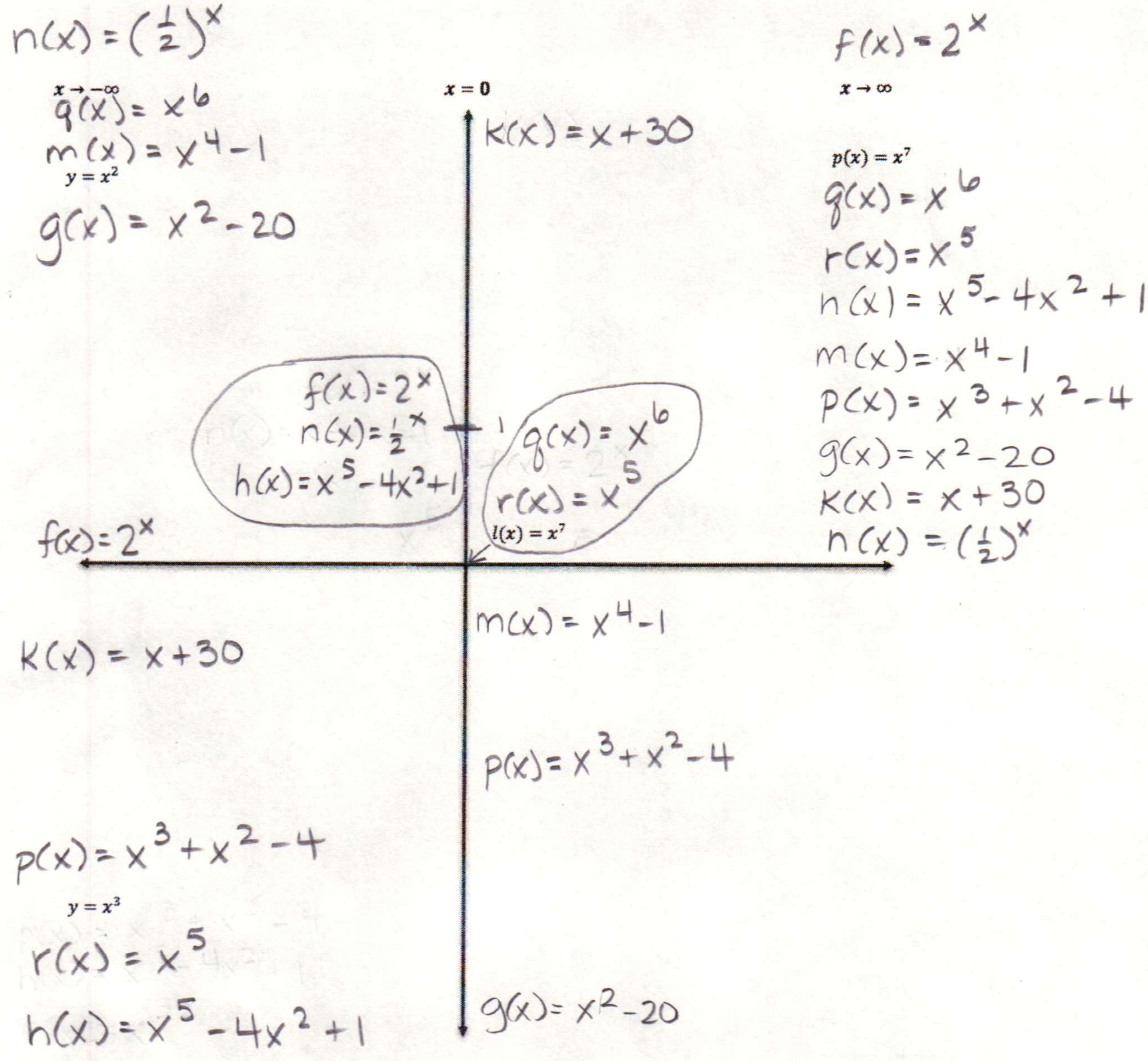
Order each function from Greatest (at the top) to Least (at the bottom) on the provided graph when:

$x = -\infty$ $\qquad\qquad$ $x = 0$ $\qquad\qquad$ $x = \infty$

Some sample equations are on the graph already.

Functions:

$f(x) = 2^x$	$p(x) = x^3 + x^2 - 4$	$g(x) = x^2 - 20$
$h(x) = x^5 - 4x^2 + 1$	$k(x) = x + 30$	$m(x) = x^4 - 1$
$r(x) = x^5$	$n(x) = \left(\frac{1}{2}\right)^x$	$q(x) = x^6$



What do you notice? Any similarities in each quadrant?

odd degree are in quadrant I & III
even degree are in quadrant I & II
the constant at the end of a polynomial is the y-intercept

Write a general rule to help you remember what each time of graph does.

odd degree goes in opposite directions
even degree goes in the same direction

Use what you learned to determine the end behavior of each function.

a. $f(x) = 3 + 2x$

Function type: linear

End behavior as $x \to -\infty$: down

End behavior as $x \to \infty$: up

b. $f(x) = x^4 - 16$

Function type: quartic

End behavior as $x \to -\infty$: up

End behavior as $x \to \infty$: up

c. $f(x) = 3^x$

Function type: exponential

End behavior as $x \to -\infty$: approaches x-axis

End behavior as $x \to \infty$: up

d. $f(x) = x^3 + 2x^2 - x + 5$

Function type: cubic

End behavior as $x \to -\infty$: down

End behavior as $x \to \infty$: up

e. $f(x) = -2x^3 + 2x^2 - x + 5$

Function type: cubic

End behavior as $x \to -\infty$: up

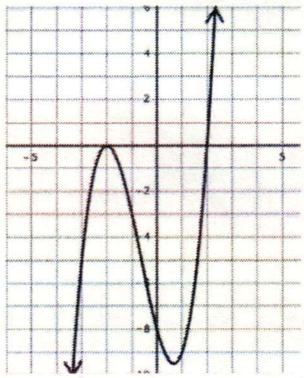
End behavior as $x \to \infty$: down

f. $f(x) = \log_2 x$

Function type: logarithm

End behavior as $x \to -\infty$: x cannot $\to -\infty$

End behavior as $x \to \infty$: up, slowly g.

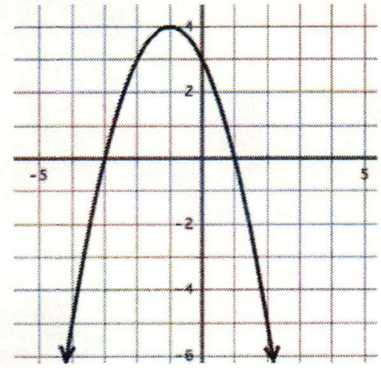


Function type: cubic

End behavior as $x \to -\infty$: down

End behavior as $x \to \infty$: up

h.



Function type: quadratic

End behavior as $x \to -\infty$: down

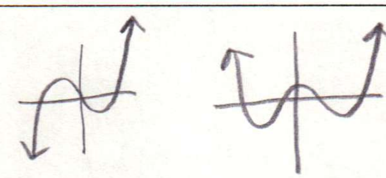
End behavior as $x \to \infty$: down

What happens if the leading coefficient is negative? it gets reflected over the x-axis

Does this change your rule you wrote earlier?

you must pay attention to which way it goes

Vocabulary

Word	Meaning/Notation	Example
End Behavior	which direction the graph goes as you get far away from the origin	

x^3 x^4